

DIFFRACTION OF A DIVERGENT BEAM

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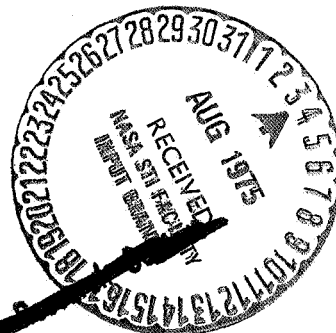
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ABSTRACT

Diffraction of a diverging light beam by a plane grating and focusing with a spherical concave mirror causes the focal distance to vary with wavelength and the linear dispersion at the focal surface to be a strong function of the grating-to-mirror distance. A simple analytic method is discussed which explains these observations.

Facility Form 802

X66-37052	N67-87179
(ACCESSION NUMBER)	(THRU)
20	2A
(PAGES)	(CODE)
TMX-57325	23
(NASA CR OR TMX OR AD NUMBER)	(CATEGORY)



(NASA-TM-X-57325) DIFFRACTION OF A
DIVERGENT BEAM (NASA) 20 p

N76-70905

INTRODUCTION

In connection with tests at Ames Research Center an f/60 scanning spectrometer was used to measure radiation emitted from the gas cap of models in a ballistic range. An f/60 beam diverges at approximately 1° and is, therefore, nearly parallel. The suggestion was made that it might be possible to simplify the light-collection system by eliminating the collimating mirror and illuminating the grating with the diverging beam. A brief literature search failed to find any references on this subject, so a simple experiment to investigate the diffraction of a diverging beam was set up in the laboratory.*

Figure 1 is a sketch of the instrument setup. A slit was placed in front of an Hg arc source, the divergent beam

*After completing this work it was discovered that a monochromator whose operation is based on the diffraction of a diverging beam has been built by Mr. James Chisholm of Bausch and Lomb Inc.¹ A description of this instrument was given at the Pittsburg Conference of the Society for Analytical Chemists, March 8, 1962. However, this paper did not contain a discussion of equation describing the optical phenomena involved.

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fell directly onto the grating, and the diffracted monochromatic beams were focused by a spherically concave mirror. The positions and orientations of the various components were changed and effects on the focused spectrum were noted. These observations revealed two major effects of diffracting a diverging beam: (1) the focal distance changed with wavelength, and (2) the linear dispersion at the focal surface increased as the grating-to-mirror distance was reduced. After these effects were noted a simplified analysis of the system explained the observed results.

CHROMATIC EFFECT INTRODUCED BY THE GRATING

The change in focal distance with wavelength was somewhat surprising since the focusing element was a mirror. Therefore, this chromatic effect was assumed to be introduced by the grating. Figure 2 shows the grating and the diverging beams. The incident beam has an incidence angle i and a divergence Δi . Each of the monochromatic diffracted beams has a mean diffraction angle $\bar{\theta}$ and a divergence, $\Delta\theta$, from a virtual source. The analysis was simplified by assuming the incident beam to be parallel over a few lines on the grating, permitting the grating equation to be used at any point on the grating.

Diffraction of a divergent beam may differ from diffraction of a collimated beam in two important ways. A change in grating dispersion may occur due to an important nonlinear change in diffraction angle as the incidence angle changes by Δi . However, the analysis showed that if the divergence of the incident beam is small, approximately 1° or less, and if the mean diffraction angle $\bar{\theta}$ is less than 60° , then $\bar{\theta}$ is nearly equal to the diffraction angle for a collimated beam and, hence, under these conditions grating dispersion is nearly unchanged from that of a collimated beam. The second effect is that $\Delta\theta$ may be a function of the mean diffraction angle $\bar{\theta}$ (wavelength). As the distance from the grating to the virtual source is controlled by $\Delta\theta$, if $\Delta\theta$ is not constant across the spectrum, the focal distance will vary when the beams are focused. This effect is analyzed below.

Figure 2 illustrates the diffraction angles from the lower and upper edges of the grating, θ_1 and θ_2 . For small Δi , $\theta_2 = \theta_1 + (d\theta/di)\Delta i$, yielding $\Delta\theta/\Delta i = d\theta/di$. As noted above, grating dispersion is effectively unchanged by using a slightly divergent beam; therefore, $d\theta/di$ can be evaluated from the grating equation. Differentiating the grating equation gives, for small Δi , $\Delta\theta/\Delta i \approx \cos i / \cos \theta$. This

expression is shown in Fig. 3, plotted against the mean diffraction angle $\bar{\theta}$. The dashed curves show values of $\Delta\theta/\Delta i$ for $\Delta i = 10^\circ$ computed from the diffraction angles at the lower and upper edges of the grating. Clearly the approximation $\cos i/\cos \theta$ for $\Delta\theta/\Delta i$ is very good.

Figure 3 shows that the divergence of the diffracted beam, $\Delta\theta$, may be less than or considerably greater than the divergence of the incident beam, Δi . Note that $\Delta\theta$ is a minimum and that the rate of change of $\Delta\theta$ with $\bar{\theta}$ is zero when $\bar{\theta} = 0$. Therefore to minimize changes in the focal distance, the incidence angle should be computed from the grating equation by setting the diffraction angle equal to zero and using the central wavelength of the region of interest.

DISPERSION EFFECT INTRODUCED BY GRATING-MIRROR COMBINATION

The second phenomenon observed by the laboratory measurements was a large increase in the linear dispersion at the focal surface as the grating-to-mirror distance was decreased. In the prior section it was shown that grating dispersion is essentially unchanged by using a slightly divergent beam. Therefore, the dispersion effect noted was assumed to be caused by the focusing of diverging beams coming from different

virtual sources. This assumption was verified by applying the ray-tracing technique to the grating-mirror combination. Figure 4 shows the model used in this analysis. Shown are the source, the divergent incident beam, the grating, two diffracted beams, the focusing mirror, and the focal surface. To simplify the geometry involved in the analysis, the grating and its normal were assumed to be located on the mirror axis.

The analysis proceeded by finding the focal points for $\theta = +1/2^\circ$ and $-1/2^\circ$ by using the rays from the edges of the grating. The distance between these two focal points along the focal surface, Δl , was taken as a measure of the linear dispersion. Δl was computed for various values of grating-to-mirror distance, L , grating width, W , and divergence of the diffracted beam, $\Delta\theta$. The values of Δl for $\Delta\theta = 0$, a collimated system, are denoted by Δl_0 .

The ratio $\Delta l/\Delta l_0$ is plotted in Fig. 5 as a function of the grating-to-mirror distance divided by the mirror focal length. Curves of constant $\Delta\theta$ are shown and grouped for constant grating size. The reason grating size affects the dispersion can be explained by referring back to Fig. 2. For fixed divergence $\Delta\theta$ the distance from the grating to the virtual source will be a function of grating width.

Hence, the larger the grating, the greater the distance from the mirror to the virtual source and the beam will appear to the mirror to be more parallel.

Figure 5 shows that the linear dispersion at the focal surface can be much less or much greater than that for a collimated system. Note the interesting result that all of the curves pass through the point (1,1). This means that if the grating-to-mirror distance is equal to the focal length of the mirror, the dispersion at the focal surface will be equal to that for a collimated system. However, if greater dispersion is desired, the mirror should be located as close to the grating as possible.

In the prior section it was shown that $\Delta\theta$ is a function of wavelength, and in Fig. 5 it is shown that at any grating-to-mirror distance other than $L/f = 1.0$ the dispersion varies with $\Delta\theta$. The effect of changes in $\Delta\theta$ on nonlinearity of the spectrum has been evaluated for one case with $L/f = 0.67$ and is shown in Fig. 6. In this figure the reciprocal dispersion is plotted against wavelength for the system shown on the figure. Note that the grating-to-mirror distance, 500 mm, is less than the focal length of the mirror. From Fig. 5 then, we expect the dispersion to be greater than that for a collimated system.

This is shown here by the fact that the reciprocal dispersion curve for $\Delta i = 1^\circ$ lies below the curve for $\Delta i = 0^\circ$. However, we note that in this case the nonlinearity of the spectrum is nearly the same for both cases. This means that the main contribution to the nonlinearity comes from the grating equation and the additional amount from the change in $\Delta\theta$ across the spectrum is insignificant. It must be emphasized that this result is for one particular case. The effect may be much greater for some other example.

CONCLUSION

In conclusion, grating instruments can be built to diffract a diverging beam without great complexity if the f-number of the incident beam is reasonably large, say > 50 . However, the designer must be careful to choose the proper orientation of the components to minimize changes in focal distance with wavelength and to benefit from the increased dispersion possible without introducing a large nonlinearity in the spectrum.

REFERENCE

1. J. J. Chisholm, Monochromator of the Type Having a Plane Grating Therein. Patent no. 3,216,313 (1965).

FIGURE LEGENDS

Fig. 1.- Divergent beam spectrograph.

Fig. 2.- Divergent beam diffraction.

Fig. 3.- Variation of $\Delta\theta/\Delta\lambda$ with diffraction angle.

Fig. 4.- Geometric model for determining dispersion.

Fig. 5.- Variation of dispersion with grating to focusing
mirror distance.

Fig. 6.- Variation in reciprocal dispersion with wavelength.

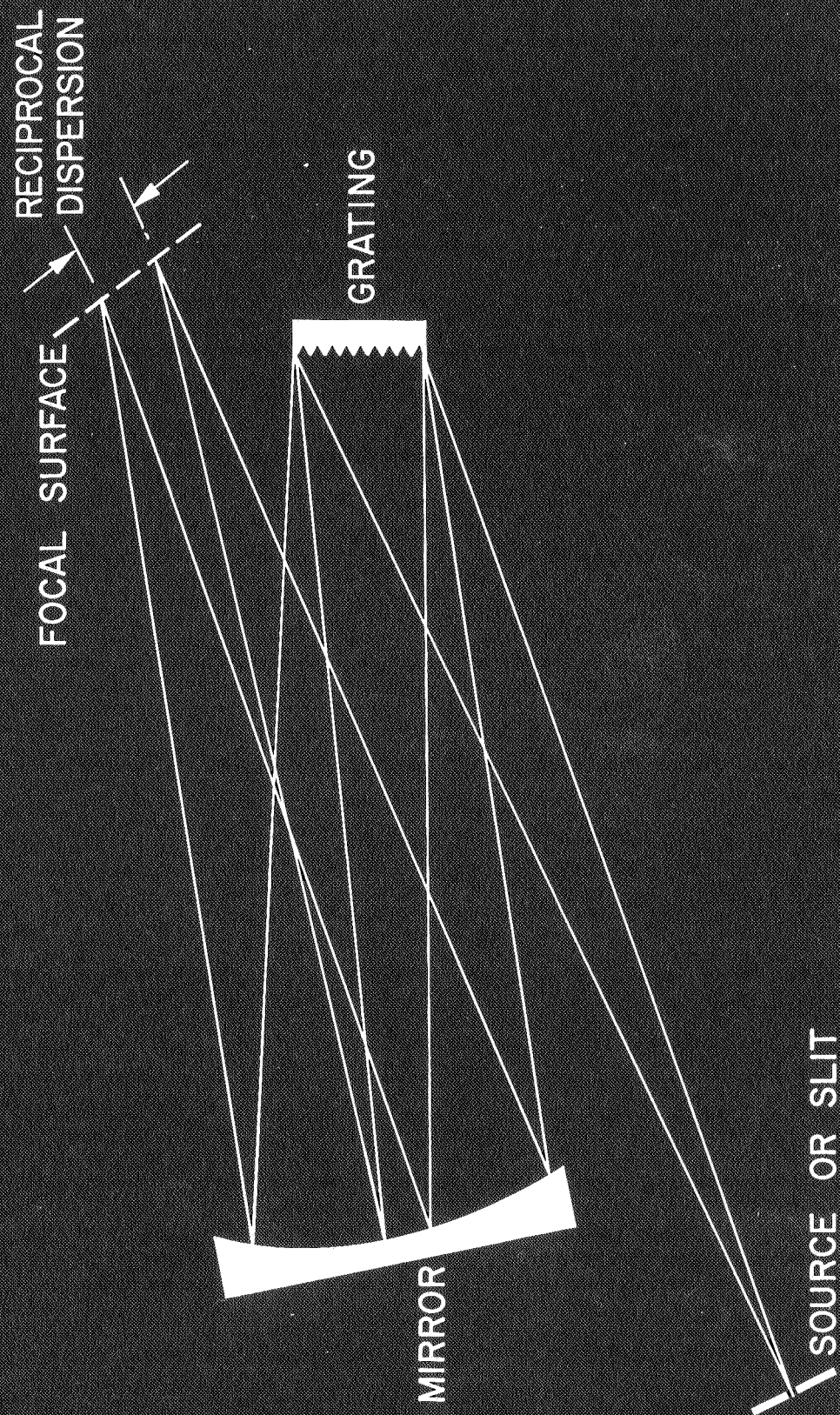


Fig. 1.- Divergent beam spectrograph.

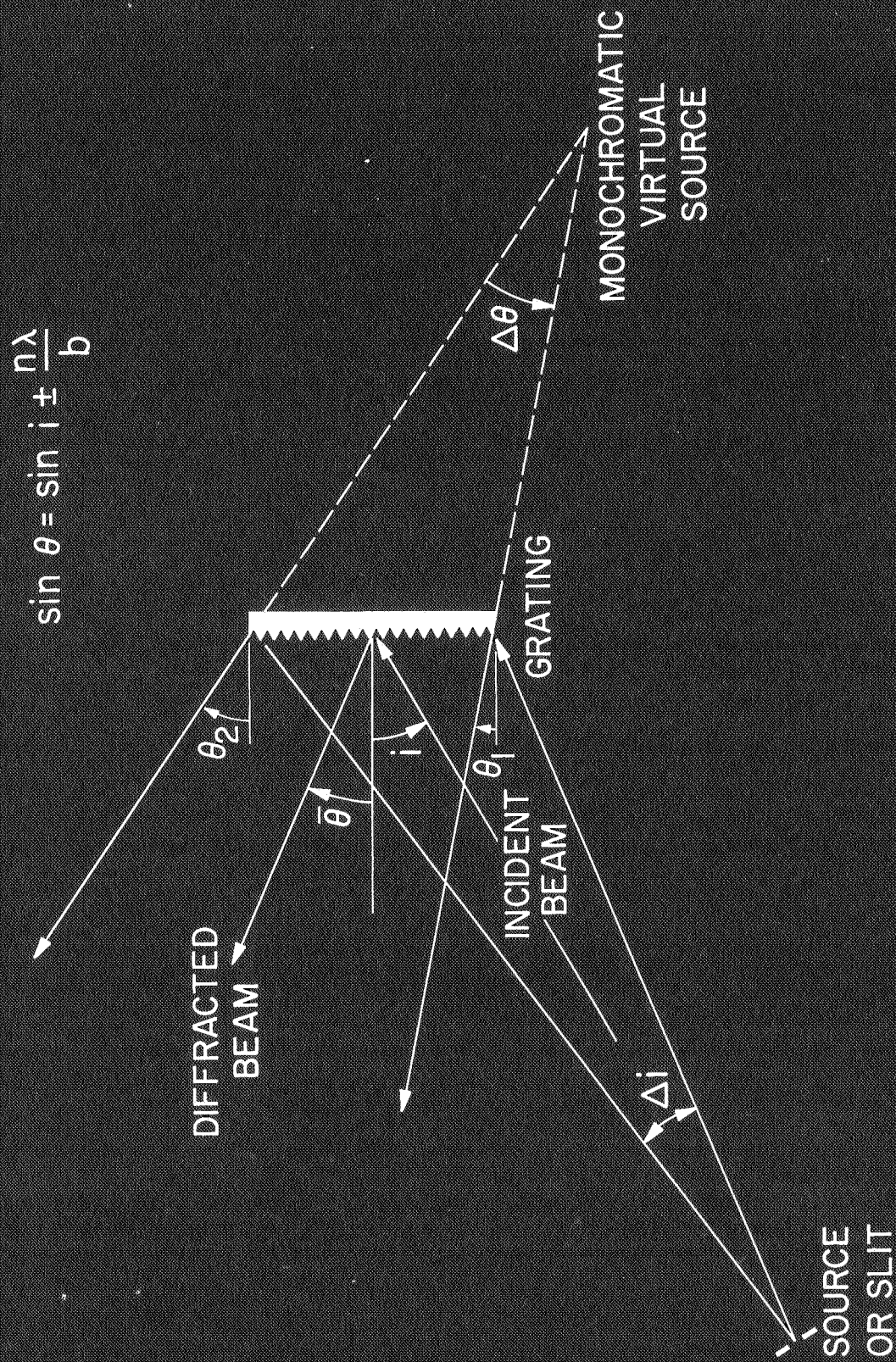


Fig. 2.- Divergent beam diffraction.

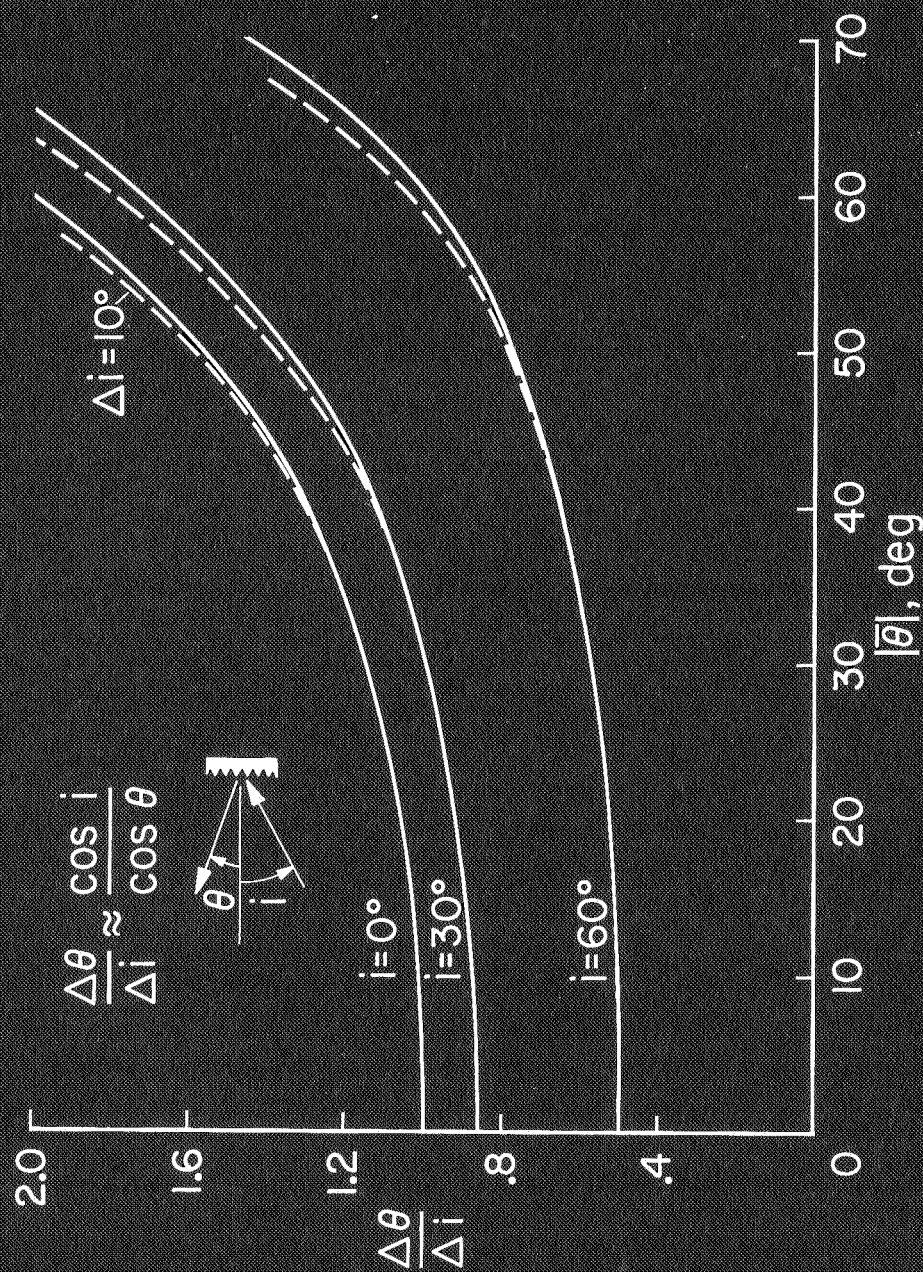


Fig. 3.- Variation of $\Delta\theta/\Delta i$ with diffraction angle.

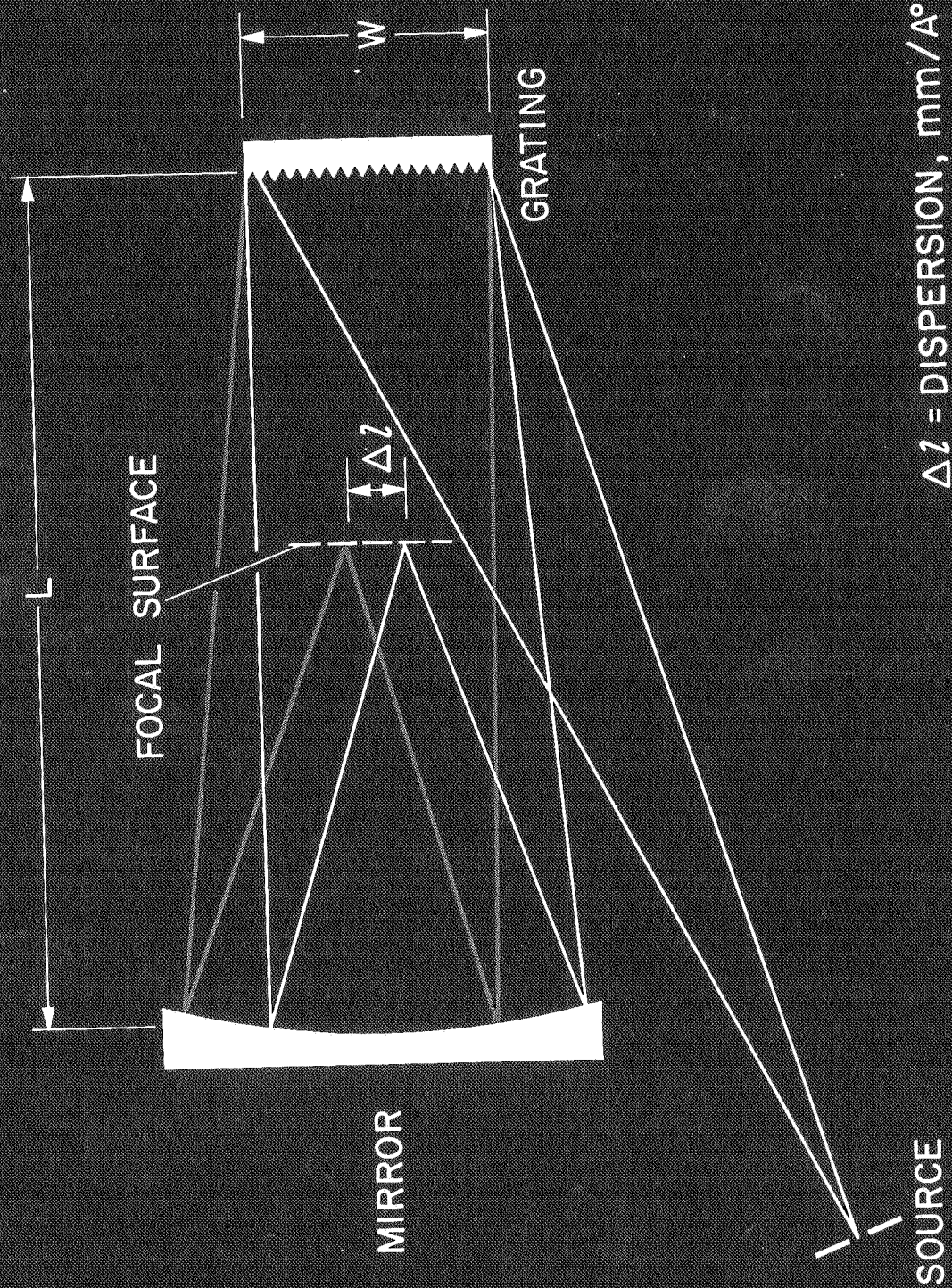


Fig. 4. - Geometric model for determining dispersion.

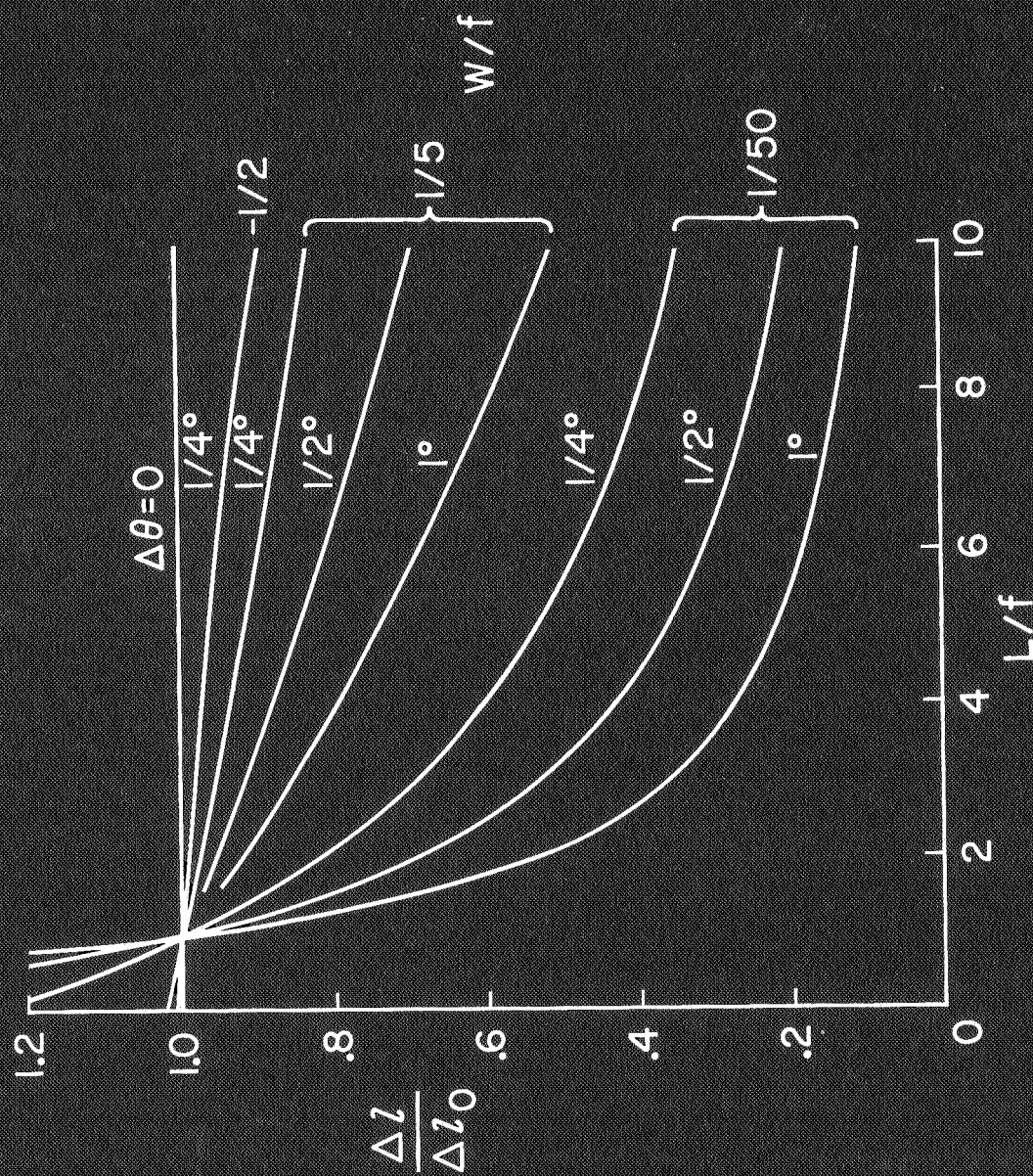


Fig. 5.- Variation of dispersion with grating to focusing mirror distance.

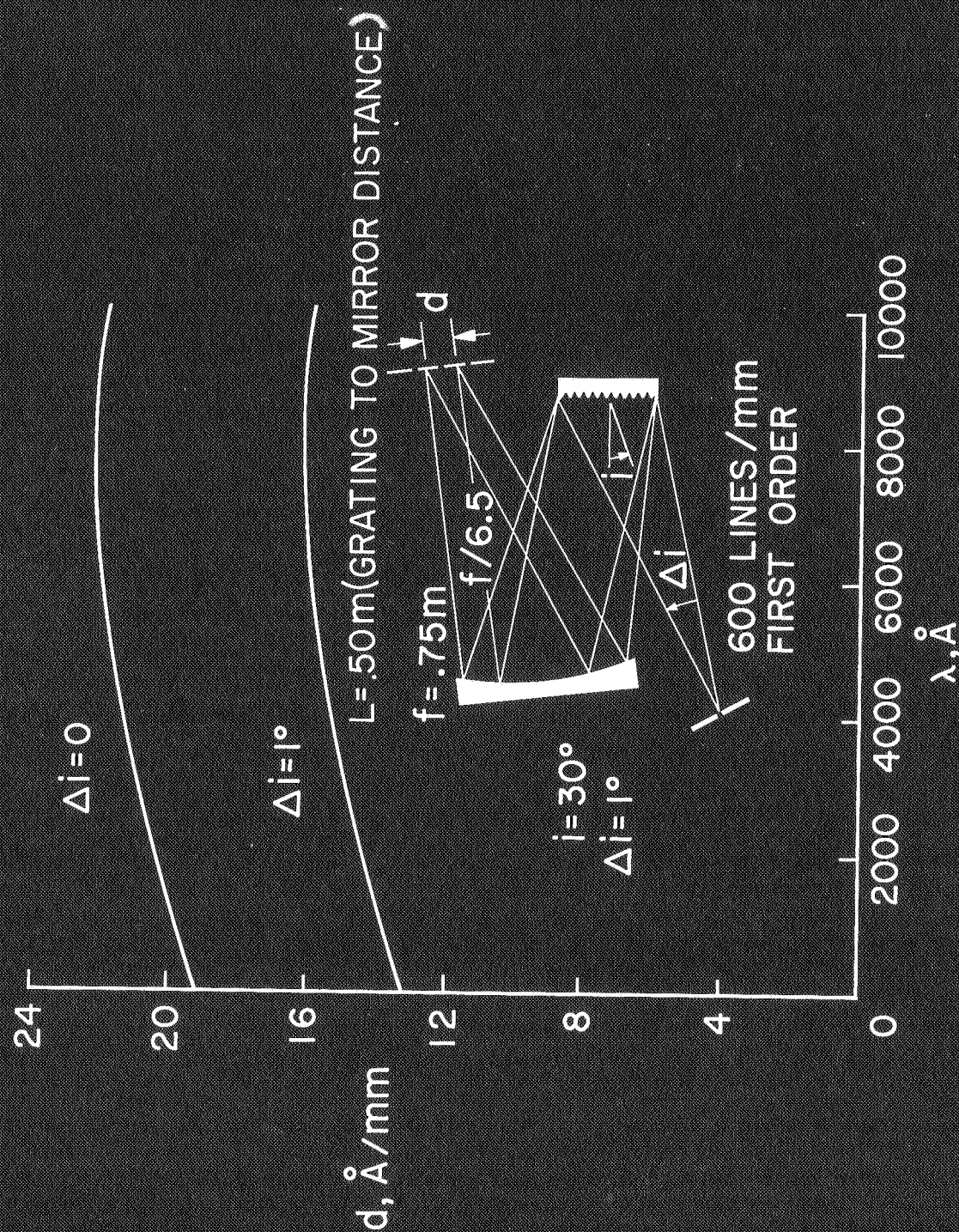


Fig. 6.- Variation in reciprocal dispersion with wavelength.